

# Charpit's Method

Method is applied to solve those equations that can't be reduced to any standard forms.

$$f(x, y, z, p, q) = 0$$

Charpit's Equations are :

$$\frac{dx}{f_p} = \frac{dy}{f_q} = \frac{dz}{pf_p + qf_q} = \frac{dp}{-(f_x + pf_z)}$$
$$= \frac{dq}{-(f_y + qf_z)}$$

$$dz = pdx + qdy \quad \text{--- (i)}$$

Integrating (i) gives complete solution

$$\text{Ques. } (x^2 - y^2) pq - xy(p^2 - q^2) - 1 = 0$$

Solution :

$$f(x, y, z, p, q) = (x^2 - y^2) pq - xy(p^2 - q^2) - 1$$

Auxiliary equations are

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$$\frac{dx}{f_p} = \frac{dy}{f_q} = \frac{dz}{pf_p + qf_q} = \frac{dp}{-(f_x + pf_z)} = \frac{dq}{-(f_y + qf_z)}$$

$$f_p = (x^2 - y^2)q - 2pxy$$

$$f_q = (x^2 - y^2)p + 2qxy$$

$$f_x = 2xpq - y(p^2 - q^2)$$

$$f_y = -2ypq - x(p^2 - q^2)$$

$$\frac{dx}{(x^2 - y^2)q - 2pxy} = \frac{dy}{(x^2 - y^2)p + 2qxy}$$

$$= \frac{dz}{pq(x^2 - y^2) - 2p^2xy + pq(x^2 - y^2) + 2q^2xy}$$

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$$= \frac{dp}{-2xpq + y(p^2 - q^2)} = \frac{dq}{2ypq + x(p^2 - q^2)}$$

Taking  $p, q, x, y$  as multipliers for first, second, fourth & fifth fractions

$$\frac{pdz + qdy + xdp + ydq}{pq(x^2 - y^2) - 2p^2xy + (x^2 - y^2)pq + 2q^2xy}$$

$$- 2x^2pq + xy(p^2 - q^2) + 2y^2pq + xy(p^2 - q^2)$$

$$\underline{Pdx + qdy + xdp + ydq} = 0$$

$$Pdx + qdy + xdp + ydq = 0$$

$$(x dp + P dx) + (y dq + q dy) = 0$$

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$$d(xP) + d(yq) = 0$$

Integrating

$$xp + yq = a$$

$$P = \frac{a - qy}{x}$$

$$(x^2 - y^2)pq - xy(p^2 - q^2) - 1 = 0$$

$$(x^2 - y^2)\left(\frac{a - qy}{x}\right)q$$

$$-xy\left(\left(\frac{a - qy}{x}\right)^2 - q^2\right) - 1 = 0$$

$$\left(\frac{a - qy}{x}\right)(qx^2 - y^2q - ay + qy^2) + q^2xy - 1 = 0$$

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$$x \left\{ \begin{array}{l} aqx^2 - q^2x^2y - ayz^2 + aqy^2 - ya^2 \\ - q^2y^3 - q^2y^2a + (-q^2y^3) + q^2xy \end{array} \right. - x = 0$$

$$ax^2q - ya^2 - x^2yq^2 + y^2aq + x^2q^2y$$

$$(ax^2 + aq^2)y = x + a^2y$$

$$q_V = \frac{x + a^2 y}{a(x^2 + y^2)}$$

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$$P = \frac{a - q_V y}{x}$$

$$(x^2 - y^2) P q_V - xy(P^2 - q_V^2) - 1 = 0$$

$$(x^2 - y^2) \left( \frac{a - q_V y}{x} \right) q_V$$

$$-xy \left[ \left( \frac{a - q_V y}{x} \right)^2 - q_V^2 \right] - 1 = 0$$

$$\left( \frac{a - q_V y}{x} \right) \left[ x^2 q_V - y^2 q_V - ya + q_V y^2 \right]$$

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$$+ xy q_V^2 - 1 = 0$$

$$(a - q_V y) (x^2 q_V - ya) + x^2 y q_V^2 - x = 0$$

$$-x^2 y q_V^2 + a x^2 q_V - ya^2 - y^2 a q_V + x^2 y q_V^2 - x = 0$$

$$(a x^2 + a y^2) q_V = x + a^2 y$$

$$q_V = \frac{x + a^2 y}{a(x^2 + y^2)}$$

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$$P = \frac{1}{x} \left( a - \left( \frac{xy + a^2 y^2}{a(x^2 + y^2)} \right) \right)$$

$$= \frac{1}{x} \left[ \frac{a^2 x^2 + a^2 y^2 - xy - a^2 y^2}{a(x^2 + y^2)} \right]$$

$$dz = pdx + q dy$$

$$= \frac{a^2 x - y}{a(x^2 + y^2)} dx + \left( \frac{x + a^2 y}{a(x^2 + y^2)} \right) dy$$

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$$= a \left( \frac{x dx + y dy}{x^2 + y^2} \right).$$

$$+ \left( \frac{x dy - y dx}{a(x^2 + y^2)} \right)$$

$$dz = a d \left( \frac{1}{2} \log(x^2 + y^2) \right)$$

$$+ \frac{1}{a} d \left( \tan^{-1} \frac{y}{x} \right)$$

Integrating

$$z = \frac{a}{2} \log(x^2 + y^2)$$

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$$+ \frac{1}{a} \tan^{-1} \left( \frac{y}{x} \right) + b$$

which gives the complete solution  
of the given equation.

\* Que. :  $Px + qy = pq$ , solve.

Sol. :  $f(x, y, z, P, q) =$   
 $Px + qy - pq = 0$ . — (1)

Auxiliary Equations are,

$$\frac{dx}{f_p} = \frac{dy}{f_q} = \frac{dz}{Pf_p + qf_q} = \frac{dp}{-(f_x + Pf_z)}$$

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$$= \frac{dq}{-(f_y + qf_z)}$$

$$\frac{dx}{x-q} = \frac{dy}{y-p} = \frac{dz}{P(x-q) + q(y-p)}$$

$$= \frac{dp}{-Pq} = \frac{dq}{-q} — (2)$$

Taking last two fractions

$$\frac{dp}{-P} = \frac{dq}{-q}$$

$$\log p = \log q + \log a$$

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$$p = qa$$

— (3)

Put  $p = qa$  in Equation (1)

$$aqx + qy = aq^2$$

$$q = \frac{y+ax}{a}$$

$$P = qa$$

$$P = a \left( \frac{y+ax}{a} \right)$$

$$P = y + ax$$

$$dz = pdx + qdy$$

$$dz = (y + ax)dx + \left( \frac{y+ax}{a} \right) dy$$

$$adz = aydx + axdx + axdy + ydy$$

$$adz = a^2x dx + ydy + a(xdy + ydx)$$

$$adz = a^2x dx + ydy + ad(xy)$$

Integrating

$$az = \frac{a^2x^2}{2} + \frac{y^2}{2} + axy + b$$

$$az = \frac{a^2x^2 + y^2 + 2axy}{2} + b$$

$$az = \frac{(ax+y)^2}{2} + b$$

which gives the complete solution  
of the given equation.